4-6 Square Roots and Cube Roots

Find each square root.

1. $\sqrt{16}$

SOLUTION:

$\sqrt{16} = 4$

ANSWER:
4

2. $-\sqrt{100}$

SOLUTION:

$-\sqrt{100} = -10$

ANSWER:
−10

3. $\pm\sqrt{81}$

SOLUTION:

$\pm\sqrt{81} = \pm 9$

ANSWER:
±9

Estimate each square root to the nearest integer.

4. $\sqrt{27}$

SOLUTION:

The first perfect square less than 27 is 25. $\sqrt{25} = 5$
The first perfect square greater than 27 is 36. $\sqrt{36} = 6$
The square root of 27 is between the integers 5 and 6. Since 27 is closer to 25 than to 36, $\sqrt{27}$ is closer to 5 than to 6.

ANSWER:
5

5. $-\sqrt{48}$

SOLUTION:

The first perfect square less than 48 is 36. $\sqrt{36} = 6$
The first perfect square greater than 48 is 49. $\sqrt{49} = 7$
The negative square root of 48 is between the integers −6 and −7. Since 48 is closer to 49 than to 36, $-\sqrt{48}$ is closer to −7 than to −6.

ANSWER:
−7

6. $\pm\sqrt{39}$

SOLUTION:
The first perfect square less than 39 is 36.
$\sqrt{36} = 6$
The first perfect square greater than 39 is 49. $\sqrt{49} = 7$
The positive and negative square roots of 39 are between the integers ±6 and ±7. Since 39 is closer to 36 than to 49, $\pm\sqrt{39}$ is closer to ±6 than to ±7.

ANSWER:
±6

7. A baseball diamond is actually a square with an area of 8100 square feet. Most baseball teams cover their diamond with a tarp to protect it from the rain. The sides are all the same length. How long is the tarp on each side?

SOLUTION:
The length of each side of the tarp is equal to the square root of the area.
$\sqrt{8100} = 90$
The tarp is 90 feet long on each side.

ANSWER:
90 ft

Find each cube root.

8. $\sqrt[3]{512}$

SOLUTION:

Since 8 • 8 • 8 = 512, the cube root of 512 is 8.

$\sqrt[3]{512} = 8$

ANSWER:

8

9. $\sqrt[3]{2197}$

SOLUTION:

Since 13 • 13 • 13 = 2197, the cube root of 2197 is 13.

$\sqrt[3]{2197} = 13$

ANSWER:

13

10. $\sqrt[3]{-1000}$

SOLUTION:

Since −10 • −10 • −10 = −1000, the cube root of −1000 is −10.

$\sqrt[3]{-1000} = -10$

ANSWER:

−10

11. $\sqrt[3]{-343}$

SOLUTION:

Since −7 • −7 • −7 = −343, the cube root of −343 is −7.

$\sqrt[3]{-343} = -7$

ANSWER:

−7

Estimate each cube root to the nearest integer.

12. $\sqrt[3]{74}$

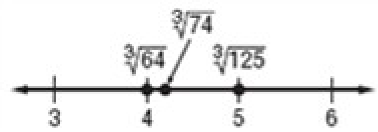
SOLUTION:

The first perfect cube less than 74 is 64. $\sqrt[3]{64} = 4$

The first perfect cube greater than 74 is 125.

$\sqrt[3]{125} = 5$

Approximate the placement of $\sqrt[3]{74}$ on a number line relative to 4 and 5. Since 74 is closer to 64 than to 125, $\sqrt[3]{74}$ will be closer to 4.



.

ANSWER:

4

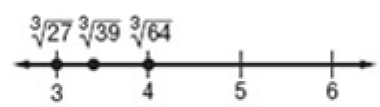
13. $\sqrt[3]{39}$

SOLUTION:

The first perfect cube less than 39 is 27. $\sqrt[3]{27} = 3$

The first perfect cube greater than 39 is 64. $\sqrt[3]{64} = 4$

Approximate the placement of $\sqrt[3]{39}$ on a number line relative to 3 and 4. Since 39 is closer to 27 than to 64, $\sqrt[3]{39}$ will be closer to 3.



ANSWER:

3

14. $\sqrt[3]{-636}$

SOLUTION:

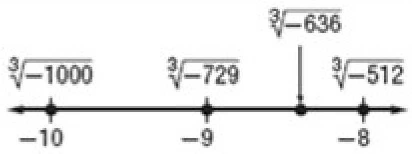
The first perfect cube less than –636 is –729.

$\sqrt[3]{-729} = -9$

The first perfect cube greater than –636 is –512.

$\sqrt[3]{-512} = -8$

Approximate the placement of $\sqrt[3]{-636}$ on a number line relative to –9 and –8. Since –636 is closer to –729 than to –512, $\sqrt[3]{-636}$ will be closer to –9.



ANSWER:

−9

15. $\sqrt[3]{-879}$

SOLUTION:

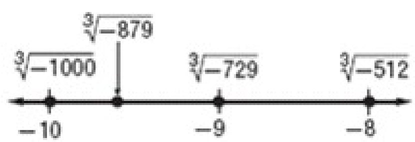
The first perfect cube less than –879 is –1000.

$\sqrt[3]{-1000} = -10$

The first perfect cube greater than –879 is –729.

$\sqrt[3]{-729} = -9$

Approximate the placement of $\sqrt[3]{-879}$ on a number line relative to –10 and –9. Since –879 is closer to –1000 than to –729, $\sqrt[3]{-879}$ will be closer to –10.



ANSWER:

−10

Find each square root.

16. $\sqrt{36}$

SOLUTION:

$\sqrt{36} = 6$

ANSWER:

6

17. $\sqrt{9}$

SOLUTION:

$\sqrt{9} = 3$

ANSWER:

3

18. $-\sqrt{169}$

SOLUTION:

$-\sqrt{169} = -13$

ANSWER:

−13

19. $-\sqrt{144}$

SOLUTION:

$-\sqrt{144} = -12$

ANSWER:

−12

20. $\pm\sqrt{-25}$

SOLUTION:

There is no real square root because no number times itself is equal to −25.

ANSWER:

no real solution

21. $\pm\sqrt{1}$

SOLUTION:

$\pm\sqrt{1} = \pm 1$

ANSWER:

±1

4-6 Square Roots and Cube Roots

Estimate each square root to the nearest integer.

22. $\sqrt{83}$

SOLUTION:

The first perfect square less than 83 is 81. $\sqrt{81} = 9$
The first perfect square greater than 83 is 100. $\sqrt{100} = 10$
The square root of 83 is between the integers 9 and 10. Since 83 is closer to 81 than to 100, $\sqrt{83}$ is closer to 9 than to 10.

ANSWER:
9

23. $\sqrt{34}$

SOLUTION:

The first perfect square less than 34 is 25. $\sqrt{25} = 5$
The first perfect square greater than 34 is 36. $\sqrt{36} = 6$
The square root of 34 is between the integers 5 and 6. Since 34 is closer to 36 than to 25, $\sqrt{34}$ is closer to 6 than to 5.

ANSWER:
6

24. $-\sqrt{102}$

SOLUTION:

The first perfect square less than 102 is 100. $\sqrt{100} = 10$
The first perfect square greater than 102 is 121. $\sqrt{121} = 11$
The negative square root of 102 is between the integers –10 and –11. Since 102 is closer to 100 than to 121, $-\sqrt{102}$ is closer to –10 than to –11.

ANSWER:
−10

25. $-\sqrt{14}$

SOLUTION:

The first perfect square less than 14 is 9. $\sqrt{9} = 3$
The first perfect square greater than 14 is 16. $\sqrt{16} = 4$
The negative square root of 14 is between the integers –3 and –4. Since 14 is closer to 16 than to 9, $-\sqrt{14}$ is closer to –4 than to –3.

ANSWER:
−4

26. $\pm\sqrt{78}$

SOLUTION:

The first perfect square less than 78 is 64. $\sqrt{64} = 8$
The first perfect square greater than 78 is 81. $\sqrt{81} = 9$
The positive and negative square roots of 78 are between the integers ±8 and ±9. Since78 is closer to 81 than to 64, $\pm\sqrt{78}$ is closer to ±9 than to ±8.

ANSWER:
±9

27. $\pm\sqrt{146}$

SOLUTION:

The first perfect square less than 146 is 144. $\sqrt{144} = 12$
The first perfect square greater than 146 is 169. $\sqrt{169} = 13$
The positive and negative square roots of 146 are between the integers ±12 and ±13. Since 146 is closer to 144 than to 169, $\pm\sqrt{146}$ is closer to ±12 than to ±13.

ANSWER:
±12

28. The table shows the heights of the tallest roller coasters at Cedar Point. Use the formula from Example 3 to determine how far a rider can see from the highest point of each ride. Round to the nearest tenth.

4-6 Square Roots and Cube Roots

Cedar Point Attractions	
Roller Coaster	Height (ft)
Mean Streak	161
Magnum XL-200	205
Millennium Force	310
Top Thrill Dragster	420

a. Millennium Force
b. Mean Streak
c. How much farther can a rider see on the Top Thrill Dragster than on the Magnum XL-200?

SOLUTION:

a.

$$\begin{aligned} d &= 1.22 \bullet \sqrt{h} \\ &= 1.22 \bullet \sqrt{310} \\ &\approx 1.22 \bullet 17.6 \\ &\approx 21.5 \end{aligned}$$

A rider can see about 21.5 miles from the highest point on the Millennium Force roller coaster.

b.

$$\begin{aligned} d &= 1.22 \bullet \sqrt{h} \\ &= 1.22 \bullet \sqrt{161} \\ &\approx 1.22 \bullet 12.7 \\ &\approx 15.5 \end{aligned}$$

A rider can see about 15.5 miles from the highest point on the Mean Streak roller coaster.

c.

Top Thrill Dragster:

$$\begin{aligned} d &= 1.22 \bullet \sqrt{h} \\ &= 1.22 \bullet \sqrt{420} \\ &\approx 1.22 \bullet 20.5 \\ &\approx 25.0 \end{aligned}$$

Magnum XL-200:

$$\begin{aligned} d &= 1.22 \bullet \sqrt{h} \\ &= 1.22 \bullet \sqrt{205} \\ &\approx 1.22 \bullet 14.3 \\ &\approx 17.5 \end{aligned}$$

A rider can see 25.0 − 17.5 or 7.5 miles farther on the Top Thrill Dragster than on the Magnum XL-200.

ANSWER:

a. 21.5 mi
b. 15.5 mi
c. 7.5 mi

Find each cube root.

29. $\sqrt[3]{-1728}$

SOLUTION:

Since −12 • −12 • −12 = −1728, the cube root of −1728 is −12.

$$\sqrt[3]{-1728} = -12$$

ANSWER:

−12

30. $\sqrt[3]{-2744}$

SOLUTION:

Since −14 • −14 • −14 = −2744, the cube root of −2744 is −14.

$$\sqrt[3]{-2744} = -14$$

ANSWER:

−14

31. $\sqrt[3]{216}$

SOLUTION:

Since 6 • 6 • 6 = 216, the cube root of 216 is 6.

$$\sqrt[3]{216} = 6$$

ANSWER:

6

32. $\sqrt[3]{1331}$

SOLUTION:

Since 11 • 11 • 11 = 1331, the cube root of 1331 is 11.

$$\sqrt[3]{1331} = 11$$

ANSWER:

11

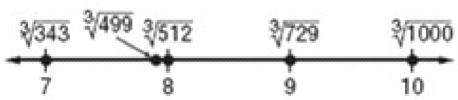
Estimate each cube root to the nearest integer. Do not use a calculator.

33. $\sqrt[3]{499}$

SOLUTION:

The first perfect cube less than 499 is 343. $\sqrt[3]{343} = 7$
The first perfect cube greater than 499 is 512.
$\sqrt[3]{512} = 8$

Approximate the placement of $\sqrt[3]{499}$ on a number line relative to 7 and 8. Since 499 is closer to 512 than to 343, $\sqrt[3]{499}$ will be closer to 8.



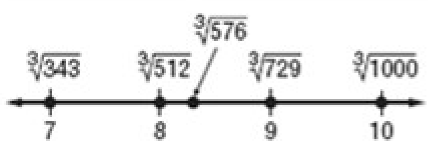
ANSWER:
8

34. $\sqrt[3]{576}$

SOLUTION:

The first perfect cube less than 576 is 512. $\sqrt[3]{512} = 8$
The first perfect cube greater than 576 is 729.
$\sqrt[3]{729} = 9$

Approximate the placement of $\sqrt[3]{576}$ on a number line relative to 8 and 9. Since 576 is closer to 512 than to 729, $\sqrt[3]{576}$ will be closer to 8.



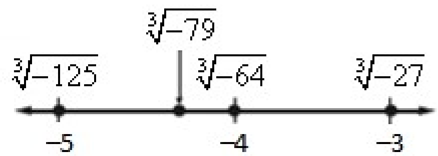
ANSWER:
8

35. $\sqrt[3]{-79}$

SOLUTION:

The first perfect cube less than –79 is –125.
$\sqrt[3]{-125} = -5$
The first perfect cube greater than –79 is –64.
$\sqrt[3]{-64} = -4$

Approximate the placement of $\sqrt[3]{-79}$ on a number line relative to –5 and –4. Since –79 is closer to –64 than to –125, $\sqrt[3]{-79}$ will be closer to –4.



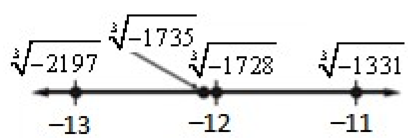
ANSWER:
–4

36. $\sqrt[3]{-1735}$

SOLUTION:

The first perfect cube less than –1735 is –2197.
$\sqrt[3]{-2197} = -13$
The first perfect cube greater than –1735 is –1728.
$\sqrt[3]{-1728} = -12$

Approximate the placement of $\sqrt[3]{-1735}$ on a number line relative to –13 and –12. Since –1735 is closer to –1728 than to –2197, $\sqrt[3]{-1735}$ will be closer to –12.



ANSWER:
–12

4-6 Square Roots and Cube Roots

37. The area of a square is 215 square centimeters. Find the length of a side to the nearest tenth. Then find its approximate perimeter.

SOLUTION:

The length of each side of the square is equal to the square root of its area.

$\sqrt{215} \approx 14.7$

A side length of the square is about 14.7 centimeters.

Use the formula $P = 4s$ to find the perimeter of the square.

$$\begin{aligned} P &= 4s \\ &= 4(14.7) \\ &= 58.8 \end{aligned}$$

The approximate perimeter of the square is 58.8 centimeters.

ANSWER:

14.7 cm; 58.8 cm

38. Order $\sqrt{77}$, −8, $-\sqrt{83}$, 9, −10, $-\sqrt{76}$, $\sqrt{65}$ from least to greatest.

SOLUTION:

Write the given integers as square roots, and then order the numbers from least to greatest.

$-8 = -\sqrt{64}$

$9 = \sqrt{81}$

$-10 = -\sqrt{100}$

$-\sqrt{100} < -\sqrt{83}, < -\sqrt{76} < -\sqrt{64} < \sqrt{65} < \sqrt{77} < \sqrt{81}$

So, the correct order is −10, $-\sqrt{83}$, $-\sqrt{76}$, −8, $\sqrt{65}$, $\sqrt{77}$, 9.

ANSWER:

−10, $-\sqrt{83}$, $-\sqrt{76}$, −8, $\sqrt{65}$, $\sqrt{77}$, 9

39. **Reason Abstractly** Write a number that completes the analogy. x^2 is to 121 as x^3 is to __?__.

SOLUTION:

121 is a perfect square of 11 since 11 • 11 = 121.
1331 is a perfect cube of 11 since 11 • 11 • 11 = 1331.

So, x^2 is to 121 as x^3 is to 1331.

ANSWER:

1331

40. **Identify Structure** Find a square root that lies between 17 and 18.

SOLUTION:

$17^2 = 289$ and $18^2 = 324$. Any number between 289 and 324 will have a square root between 17 and 18. For example, $\sqrt{300} \approx 17.3$.

ANSWER:

Sample answer: $\sqrt{300}$

41. **Persevere with Problems** Use inverse operations to evaluate the following.

a. $\left(\sqrt{246}\right)^2$

b. $\left(\sqrt{811}\right)^2$

c. $\left(\sqrt{732}\right)^2$

SOLUTION:

a. $\left(\sqrt{246}\right)^2 = 246$

b. $\left(\sqrt{811}\right)^2 = 811$

c. $\left(\sqrt{732}\right)^2 = 732$

ANSWER:

a. 246
b. 811
c. 732

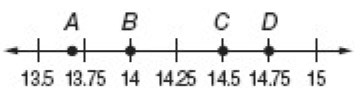
4-6 Square Roots and Cube Roots

42. **Building on the Essential Question** Describe the difference between an exact value and an approximation when finding square roots of numbers that are not perfect squares. Give an example of each.

SOLUTION:
Sample answer: The exact value of a square root is given using the square root symbol, such as $\sqrt{13}$. An approximation is a decimal value, such as $\sqrt{133} \approx 6$.

ANSWER:
Sample answer: The exact value of a square root is given using the square root symbol, such as $\sqrt{13}$. An approximation is a decimal value, such as $\sqrt{133} \approx 6$.

43. Which point on the number line best represents $\sqrt{210}$?



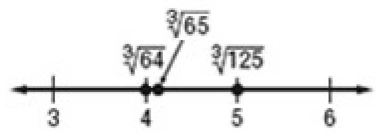
A	*A*
B	*B*
C	*C*
D	*D*

SOLUTION:
Use a calculator to estimate the square root of 210.
$\sqrt{210} \approx 14.5$
Point C best represents $\sqrt{210}$.
Choice C is the correct answer.

ANSWER:
C

44. **Short Response** Estimate the cube root of 65 to the nearest integer.

SOLUTION:
The first perfect cube less than 65 is 64. $\sqrt[3]{64} = 4$
The first perfect cube greater than 65 is 125.
$\sqrt[3]{125} = 5$
Approximate the placement of $\sqrt[3]{65}$ on a number line relative to 4 and 5. Since 65 is closer to 64 than to 125, $\sqrt[3]{65}$ will be closer to 4.



ANSWER:
4

45. The new gymnasium at Oakdale Middle School has a hardwood floor in the shape of a square. If the area of the floor is 62,500 square feet, what is the length of one side of the square floor?

F	200 ft
G	225 ft
H	250 ft
J	275 ft

SOLUTION:
The length of a side of the gymnasium floor is equal to the square root of its area.
$\sqrt{62,500} = 250$
Each side has length 250 units.
Choice H is the correct answer.

ANSWER:
H

4-6 Square Roots and Cube Roots

46. A surveyor determined the distance across a field was $\sqrt{1568}$ feet. What is the approximate distance?

A	25.6 ft
B	30.6 ft
C	39.6 ft
D	42.6 ft

SOLUTION:

Use a calculator to estimate the square root of 1568 to the nearest tenth.

$\sqrt{1568} \approx 39.6$

Choice C is the correct answer.

ANSWER:

C

Solve.

47. Joseph bought four books at a book sale. Each book cost $4.50. He paid with $20.00. How much change did he receive?

SOLUTION:

To find the cost of four books, multiply the number of books by the cost for each book.

$4 \times 4.50 = 18.00$

So, the four books cost $18.00.

To find the amount of change he received, subtract the cost of the books from the amount that Joseph paid.

$20 - 18 = 2$

So, Joseph received $2.00 in change.

ANSWER:

$2.00

48. Mrs. Tanner paid $18.00 for six boxes of pencils. How much did each box of pencils cost?

SOLUTION:

To find the cost of each box of pencils, divide the total cost by the number of boxes of pencils.

$18 \div 6 = 3$

So, each box of pencils cost $3.00.

ANSWER:

$3.00

49. Kathryn had $367.50 in her bank account. She wrote a check for $25.00, and then withdrew $50.00 in cash. She made a deposit of $100.00. How much money is in her bank account now?

SOLUTION:

Kathryn had $367.50 in her bank account.	$367.50
She wrote a check for $25.00.	$367.50 – $25.00 = $342.50
She withdrew $50.00 in cash.	$342.50 – $50.00 = $292.50
She made a deposit of $100.00.	$292.50 + $100.00 = $392.50

Kathryn has $392.50 in her bank account now.

ANSWER:

$392.50

Name the property shown by each statement.

50. $3 + 6 = 6 + 3$

SOLUTION:

The order of the numbers changed. This is the Commutative Property of Addition.

ANSWER:

Commutative Property (+)

51. $13 + 0 = 13$

SOLUTION:

When zero is added to any number, the sum is the number. This is the Additive Identity Property.

ANSWER:

Additive Identity Property

52. $(2 \bullet 5) + 6 = 6 + (2 \bullet 5)$

SOLUTION:

The order of the numbers changed. This is the Commutative Property of Addition.

ANSWER:

Commutative Property (+)

53. $(x + 3) + 9 = x + (3 + 9)$

SOLUTION:
The grouping of the numbers changed. This is the Associative Property of Addition.

ANSWER:
Associative Property (+)

54. $(y \cdot 2) \cdot 3 = y \cdot (2 \cdot 3)$

SOLUTION:
The grouping of the numbers changed. This is the Associative Property of Multiplication.

ANSWER:
Associative Property (×)

55. $28 \cdot 1 = 28$

SOLUTION:
When 1 is multiplied by any number, the product is the number. This is the Multiplicative Identity Property.

ANSWER:
Multiplicative Identity Property

56. $n + t = t + n$

SOLUTION:
The order of the variables has changed. This is the Commutative Property of Addition.

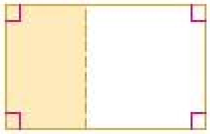
ANSWER:
Commutative Property (+)

57. $1652 \cdot 0 = 0$

SOLUTION:
When zero is multiplied by any number, the product is zero. This is the Zero Property of Multiplication.

ANSWER:
Zero Property of Multiplication

58. Suppose that four-tenths of the rectangle below is shaded.



a. What fraction of the rectangle is *not* shaded?
b. What fraction of the rectangle would still need to be shaded for half of the rectangle to be shaded?
c. If an additional $\frac{7}{15}$ of the original rectangle were to be shaded, what fraction of the rectangle would be shaded?

SOLUTION:
a. To find the fraction of the rectangle that is *not* shaded, subtract the fraction that is shaded from one whole.

$$\begin{aligned}1 - \frac{4}{10} &= \frac{1}{1} \bullet \frac{10}{10} - \frac{4}{10}\\ &= \frac{10}{10} - \frac{4}{10}\\ &= \frac{6}{10}\\ &= \frac{3}{5}\end{aligned}$$

$\frac{3}{5}$ of the rectangle is *not* shaded.

b. To find the fraction of the rectangle that would still need to be shaded, subtract the fraction that is already shaded from one-half.

$$\begin{aligned}\frac{1}{2} - \frac{4}{10} &= \frac{1}{2} \bullet \frac{5}{5} - \frac{4}{10}\\ &= \frac{5}{10} - \frac{4}{10}\\ &= \frac{1}{10}\end{aligned}$$

$\frac{1}{10}$ of the rectangle would still need to be shaded for half of the rectangle to be shaded.

c.

$$\begin{aligned}\frac{4}{10} + \frac{7}{15} &= \frac{4}{10} \bullet \frac{3}{3} + \frac{7}{15} \bullet \frac{2}{2}\\ &= \frac{12}{30} + \frac{14}{30}\\ &= \frac{26}{30}\\ &= \frac{13}{15}\end{aligned}$$

If an additional $\frac{7}{15}$ of the original rectangle were shaded, $\frac{13}{15}$ of the rectangle would be shaded.

ANSWER:

a. $\frac{3}{5}$

b. $\frac{1}{10}$

c. $\frac{13}{15}$

Find each product or quotient.

59. 8 • (–8)

SOLUTION:

The product of two integers with different signs is negative. So, 8 • (–8) = –64.

ANSWER:

–64

60. –4 • (–12)

SOLUTION:

The product of two integers with the same sign is positive. So, –4 • (–12) = 48.

ANSWER:

48

61. 40 ÷ (–5)

SOLUTION:

The quotient of two integers with different signs is negative. So, 40 ÷ (–5) = –8.

ANSWER:

–8

62. –150 ÷ (–25)

SOLUTION:

The quotient of two integers with the same sign is positive. So, –150 ÷ (–25) = 6.

ANSWER:

6